

$$Q \sin \theta \cdot \sin (60 - \theta) \cdot \sin (60 + \theta)$$

$$\sin \theta \{ \sin^2 60 - \sin^2 \theta \}$$

$$\left\{ \frac{3}{4} - \sin^2 \theta \right\}$$

$$\sin \theta \left\{ \frac{3 - 4 \sin^2 \theta}{4} \right\}$$

$$\left\{ \frac{3 \sin \theta - 4 \sin^3 \theta}{4} \right\}$$

$$= \frac{\sin 3\theta}{4}$$

$$\begin{array}{l|l} \sin(A+B) \cdot \sin(A-B) & \cos(A+B) \cdot \cos(A-B) \\ \hline = \sin^2 A - \sin^2 B & = \cos^2 B - \cos^2 A \end{array}$$

$$Q \cos \theta \cdot \cos (60 - \theta) \cdot \cos (60 + \theta)$$

$$\cos \theta \{ \cos^2 \theta - \cos^2 60 \}$$

$$\cos \theta \left\{ \cos^2 \theta - \frac{3}{4} \right\}$$

$$\cos \theta \left\{ \frac{4 \cos^2 \theta - 3}{4} \right\}$$

$$\frac{4 \cos^3 \theta - 3 \cos \theta}{4}$$

$$\frac{\cos 3\theta}{4}$$

Result:

$$1) \sin \theta \cdot \sin (60 - \theta) \cdot \sin (60 + \theta) = \frac{\sin 3\theta}{4}$$

$$2) \cos \theta \cdot \cos (60 - \theta) \cdot \cos (60 + \theta) = \frac{\cos 3\theta}{4}$$

$$3) \tan \theta \cdot \tan (60 - \theta) \cdot \tan (60 + \theta) = \tan 3\theta$$

$$G^2 B - \sin^2 A = G(A+B) \cdot G(A-B)$$

Q $1 + G^2 \frac{(A+B)}{b} - \sin^2 \frac{(A-B)}{a} - G 2A \cdot G 2B$ is Independent of A & B [T/F]

$$1 + G(A+B+A-B) \cdot G((A+B)-(A-B)) - G 2A \cdot G 2B$$

$$1 + G 2A \cancel{G 2B} - G 2A \cdot \cancel{G 2B} = 1 \quad [\text{True}]$$

Q $G^2 A + G^2(A+B) - 2GA \cdot GB \cdot G(A+B)$ is Indep. of A ?

$G(A+B)$ common thing

$$G^2 A + G(A+B) \{ G(A+B) - 2GA \cdot GB \}$$

$$\{ GA \cdot GB - \sin A \sin B - 2GA \cdot GB \}$$

$$GA - G(A+B) \{ GA \cdot GB + \sin A \sin B \}$$

$$G^2 A - G(A+B) G(A-B) = \underline{G^2 A} - (G^2 B - \underline{\sin^2 A}) = 1 - G^2 B = \sin^2 B$$

formula used?

$G^2 A + \sin^2 A = 1$ [True]
Ind. of A

Q $\frac{\sin(\alpha-\beta) \cdot \sin(\alpha+\beta)}{1 - \tan^2 \alpha \cdot \cot^2 \beta}$ Ind. v. f. α & β ?
 $\rightarrow$ Sin Cos A B data then LCM

$$\frac{\sin(\alpha-\beta) \cdot \sin(\alpha+\beta) \times \{\cos^2 \alpha \cdot \sin^2 \beta\}}{\cos^2 \alpha \cdot \sin^2 \beta - \sin^2 \alpha \cdot \cos^2 \beta} \rightarrow \text{Prime}(a^2 - b^2)$$

$$\cos^2 \alpha \cdot \sin^2 \beta - \sin^2 \alpha \cdot \cos^2 \beta \rightarrow \text{Prime}(a^2 - b^2)$$

$$\sin(\alpha+\beta) \cdot \sin(\alpha-\beta) \times \cos^2 \alpha \cdot \sin^2 \beta$$

$$(\cos \alpha \sin \beta - \sin \alpha \cos \beta)(\cos \alpha \sin \beta + \sin \alpha \cos \beta)$$

$$\sin(A-B) \leftarrow \frac{\sin(\alpha+\beta) \cdot \sin(\alpha-\beta) \cdot \cos^2 \alpha \cdot \sin^2 \beta}{\cos^2 \alpha \cdot \sin^2 \beta - \sin^2 \alpha \cdot \cos^2 \beta}$$

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formul.

$$- \sin(\alpha-\beta) \sin(\alpha+\beta)$$

dependant on
 α & β

$$\underline{15^\circ / 75^\circ}$$

$$(1) \sin 15^\circ = \sin(45^\circ - 30^\circ)$$

$$= \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ$$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2} = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

$$(2) \cos 15^\circ = \cos(45^\circ - 30^\circ)$$

$$= \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ$$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

$$(3) \tan 15^\circ = \frac{\sin 15^\circ}{\cos 15^\circ} = \frac{\sqrt{3}-1}{\sqrt{3}+1} = \underline{2-\sqrt{3}}$$

$$(4) \cot 15^\circ = \frac{\sqrt{3}+1}{\sqrt{3}-1} = 2+\sqrt{3}$$

Result

$$\sin 15^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}} = \cos 75^\circ$$

$$\cos 15^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}} = \sin 75^\circ$$

$$\tan 15^\circ = 2-\sqrt{3} = \cot 75^\circ$$

$$\cot 15^\circ = 2+\sqrt{3} = \tan 75^\circ$$

half Angle

all half angle formulas
are made by 4 formulas.

$$\cos \theta = \sqrt{\frac{1+\cos 2\theta}{2}}, \quad \sin \theta = \sqrt{\frac{1-\cos 2\theta}{2}}$$

$$\tan \theta = \frac{1-\cos 2\theta}{\sin 2\theta}, \quad \cot \theta = \frac{1+\cos 2\theta}{\sin 2\theta}$$

$$\boxed{7\frac{1}{2}^\circ / 22\frac{1}{2}^\circ}$$

$$\textcircled{1} \cos 22\frac{1}{2}^\circ = ? / \cos \frac{\pi}{8} = ?$$

$$\cos \frac{\pi}{8} = \sqrt{\frac{1+\cos \frac{\pi}{4}}{2}}$$

$$= \sqrt{\frac{1+\frac{1}{\sqrt{2}}}{2}} = \sqrt{\frac{\sqrt{2}+1}{2\sqrt{2}}}$$

Q If $\cot \left[\frac{\pi}{24} \right] = \sqrt{a} + \sqrt{b} + \sqrt{c} + \sqrt{d}$, $a > b > c > d$.
 then $a - b + c - d = ?$

$$\cot 7\frac{1}{2}^\circ = \frac{1 + \cos 15^\circ}{\sin 15^\circ} = \frac{1 + \frac{\sqrt{3}+1}{2\sqrt{2}}}{\frac{\sqrt{3}-1}{2\sqrt{2}}}$$

$$= \frac{2\sqrt{2} + \sqrt{3} + 1}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1}$$

$$= \frac{2\sqrt{6} + \textcircled{3} + \sqrt{3} + 2\sqrt{2} + \sqrt{3} + \textcircled{1}}{2}$$

$$= \frac{2\sqrt{6} + 2\sqrt{3} + 2\sqrt{2} + 4}{2}$$

$$= \sqrt{6} + \textcircled{2} + \sqrt{3} + \sqrt{2} = \sqrt{1} + \sqrt{4} + \sqrt{3} + \sqrt{2}$$

$$\cot \theta = \frac{1 + \cos 2\theta}{\sin 2\theta}$$

$$a - b + c - d = 6 - 4 + (3 - 2) = 3$$

Q If $f(\theta) = \frac{(1 - \sin 2\theta) + \cos 2\theta}{2 \cos 2\theta}$ then.

find $8 f(11^\circ) \cdot f(34^\circ)$

$$f(\theta) = \frac{(\cos \theta - \sin \theta)^2 + (\cos^2 \theta - \sin^2 \theta)}{2(\cos^2 \theta - \sin^2 \theta)}$$

$$= \frac{(\cancel{\cos \theta - \sin \theta})(\cos \theta - \cancel{\sin \theta} + \cos \theta + \cancel{\sin \theta})}{2(\cancel{\cos^2 \theta - \sin^2 \theta})(\cos \theta + \sin \theta)}$$

$$= \frac{2 \cos \theta}{2(\cos \theta + \sin \theta)} = \frac{1}{1 + \tan \theta}$$

(cos θ divide)

Concept $\rightarrow A + B = 45^\circ$

then $(1 + \tan A)(1 + \tan B) = 2$

$8 f(11^\circ) f(34^\circ)$

$11 + 34 = 45^\circ$

$$8 \times \frac{1}{(1 + \tan 11^\circ)} \times \frac{1}{(1 + \tan 34^\circ)}$$

$$= 8 \times \frac{1}{2} = 4$$

$$A+B=45^\circ$$

$$\tan(A+B)=1$$

$$\frac{\tan A + \tan B}{1 - \tan A \tan B} = 1$$

$$\tan A + \tan B = 1 - \tan A \tan B$$

$$(1 + \tan A) + \tan B + \tan A \tan B = 1 + 1$$

$$(1 + \tan A) + \tan B(1 + \tan A) = 2$$

$$(1 + \tan A)(1 + \tan B) = 2$$

$$Q \sqrt{2 + \sqrt{2 + 2\cos 4\theta}} = ? ; 3\pi < 4\theta < 4\pi$$

$$\sqrt{2 + \sqrt{2(1 + \cos 4\theta)}}$$

$$\sqrt{2 + \sqrt{2 \times 2\cos^2 2\theta}}$$

$$\sqrt{2 + 2|\cos 2\theta|}$$

$$\sqrt{2 + 2\cos 2\theta}$$

$$\sqrt{2(1 + \cos 2\theta)}$$

$$\sqrt{2 \times 2\cos^2 \theta} = 2|\cos \theta|$$

$$= -2\cos \theta$$

$$1 + \cos 2\theta = 2\cos^2 \theta$$

$$1 + \cos 4\theta = 2\cos^2 2\theta$$

$$\frac{3\pi}{2} < 2\theta < 2\pi$$



$$\frac{3\pi}{4} < \theta < \pi$$



$$Q \quad (4 \cot^2 9^\circ - 3)(4 \cot^2 27^\circ - 3) = \tan \theta + \tan \theta = ?$$

$$\left(\frac{4 \cot^3 9^\circ - 3 \cot 9^\circ}{\cot 9^\circ} \right) \times \left(\frac{4 \cot^3 27^\circ - 3 \cot 27^\circ}{\cot 27^\circ} \right) = \tan \theta$$

$$\frac{\cancel{\cot(3 \times 9^\circ)}}{\cot 9^\circ} \times \frac{\cancel{\cot 3 \times 27^\circ}}{\cancel{\cot 27^\circ}} = \tan \theta$$

$$\frac{\cot 81^\circ}{\cot 9^\circ} = \tan \theta \Rightarrow \frac{\tan 9^\circ}{\cot 9^\circ} = \tan \theta$$

$$\Rightarrow \tan \theta = \tan 9^\circ$$

$$\Rightarrow \theta = 9^\circ$$

$$\boxed{\cot 3\theta = 4 \cot^3 \theta - 3 \cot \theta}$$

$$\tan 3\theta = 3 \tan \theta - 4 \tan^3 \theta$$

$$Q \quad \sin 20^\circ \cdot \sin 40^\circ \cdot \boxed{\sin 60^\circ} \cdot \sin 80^\circ = ?$$

$$\frac{\sqrt{3}}{2} \times \sin 20^\circ \cdot \sin 40^\circ \cdot \sin 80^\circ$$

$$\frac{\sqrt{3}}{2} \times \sin 20^\circ \cdot \sin(60^\circ - 20^\circ) \cdot \sin(60^\circ + 20^\circ)$$

$$\frac{\sqrt{3}}{2} \times \frac{\sin(3 \times 20^\circ)}{4} = \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{8} = \frac{3}{16}$$

$$Q \quad \frac{\cos 20^\circ + 2 \boxed{\sin 70^\circ} \boxed{\sin 50^\circ} \sin 10^\circ}{\sin^2 80^\circ} = ?$$

$$\frac{\cos 20^\circ + 2 \times \sin 10^\circ \cdot \sin(60-10^\circ) \cdot \sin(60+10^\circ)}{\sin^2 80^\circ}$$

$$\frac{\cos 20^\circ + 2 \times \frac{\sin(3 \times 10^\circ)}{4}}{\sin^2 80^\circ} = \frac{\cos(20^\circ) + 1}{\sin^2 80^\circ}$$

$$= \frac{2 \cos^2 10^\circ}{\sin^2 90^\circ} = \frac{2 \cos^2 10^\circ}{1} = 2$$

$$1 + \cos 2\theta = 2 \cos^2 \theta$$

$$Q \quad \tan 6^\circ \cdot \tan 42^\circ \cdot \tan 66^\circ \cdot \tan 78^\circ = ?$$

$$\{\tan 6^\circ \cdot \tan 66^\circ\} \{\tan 42^\circ \cdot \tan 78^\circ \dots\}$$

$$\left\{ \frac{\tan 6^\circ \cdot \tan(60+6^\circ) \cdot \tan(60-6^\circ)}{\tan 54^\circ} \right\} \left\{ \frac{\tan(60-18^\circ) \cdot \tan(60+18^\circ) \cdot \tan 18^\circ}{\tan 18^\circ} \right\}$$

$$\frac{\cancel{\tan 6^\circ} \cdot \cancel{\tan 54^\circ}}{\cancel{\tan 54^\circ}} \times \frac{\cancel{\tan 60^\circ} \cdot \cancel{\tan 18^\circ}}{\cancel{\tan 18^\circ}} = 1$$

Ex 15, 16, 17
(Basic)